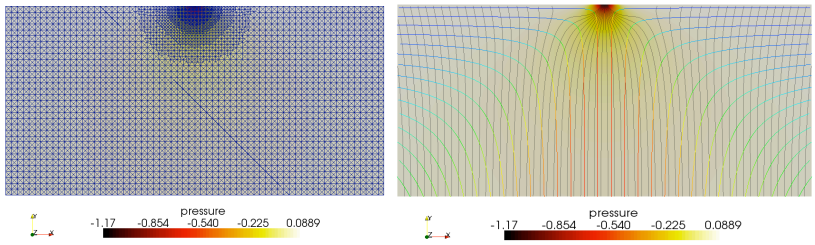
MADDs-FP – Flexible multi-physics models for Magma Dynamics using advanced computational libraries

Marc Spiegelman, Applied Math/LDEO, Columbia University, Gideon Simpson, Univ. of Toronto

Magma Dynamics is one of the more challenging multi-physics problems in solid Earth Science, requiring a consistent coupling the flow of a low viscosity fluid or melt with large scale solid deformation (e.g. mantle convection or lithospheric deformation). The coupled system is non-linear, can be highly time dependent and is often unstable to the formation of coherent small scale features such as magma waves or localized channels. To explore, these processes and understand how they affect observables and the large scale flow requires computational software that allows flexible composition of a range of models from simple process models to more realistic regional plate-boundary models, as well as providing a wide choice in solver technology. Fortunately, recent advances in computational libraries such as the FEniCS project (www.fenics.org), PETSc (www.mcs.anl.gov/petsc/) and deal.II (www.dealii.org) provide such functionality.

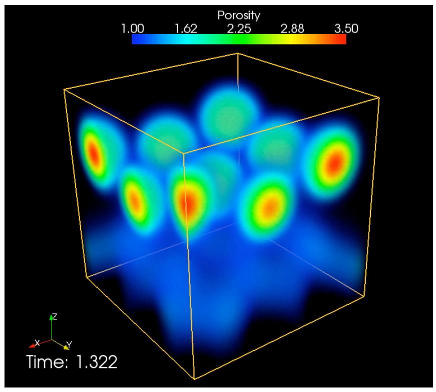
As part of the Magma Dynamics component of CIG, I have developed a new set of hybrid FEniCS/PETSc codes that implement the Magma Dynamics Demonstration Suite of benchmarks (i.e. MADDs-FP), and are available as a mercurial repository from CIG (hg clone http://geodynamics.org/hg/magma/3D/MADDs-FP). Key features used from FEniCS includes a high level language for describing the weak form of general PDE's (Unified Form Language UFL), a Form Compiler (FFC) for automatic generation of optimised C++ code, and a high-level interface for evaluating discrete functions (Dolfin) which allows easy development of semi-Lagrangian methods on fully unstructured meshes. These codes allow full use of PETSc's linear and non-linear solvers, including FieldSplit Block pre-conditioners for physics-based preconditioning. The actual choice of solver can be chosen at run time (or changed as the problem evolves). Specific Codes and Features include:



Left: mesh and pressure field from MADDs-1 Stokes solver.
Right: fluid and solid flow lines for MADDs-2 on pressure field

MADDs-1: Isoviscous Stokes flow for a mid-ocean ridge corner flow boundary condition using Taylor-Hood (P2-P1) elements on unstructured meshes and implementing optimal fieldsplit iterative solvers for iso-viscous stokes (e.g. Elman, Wathen etc) as well as singular direct solvers in PETSc.
MADDs-2: a pyDolfin post-processing script to calculate the melt-velocity field given a uniform permeability, no melting and the solid velocity and pressure field from MADDs-1. This script also demonstrates how to project velocities and pressures onto non-trivial vector-valued finite element spaces (BDM) for calculating a flux-conservative melt flow field.

MADDs-4: a full featured 2 and 3-D magmatic solitary wave Benchmark code. These codes include separate python routines for calculating spectrally accurate initial conditions for solitary waves in 1,2 and 3 Dimensions using Sinc collocation methods (Simpson & Spiegelman, 2009 in prep).



Instability of 1-D solitary wave in 3-D calculated from MADDs-4.

MADDs-5a: A 2-D time-dependent ridge melting and transport code that imports solutions from MADDs-1 for solid flow and calculates the consistent generation and transport of melt assuming adiabatic decompression melting. This model uses a new regularization of the magma equations which is easily composed in FEniCS, is fully time-dependent and uses time-adaptive implicit newton-solvers with semi-lagrangian advection and physics-based preconditioners. Figure shows porosity/0.4%, contours of melting rate filed and melt flux for time-dependent run with moderate (1e20 Pa s and large shear viscosity 5e20 Pas) both demonstrating enhanced melt flow to the ridge axis (box depth is 90 km).