

Global spectral flow code development and benchmark plan

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As a result of our discussions, your comments, and several generous offers to contribute codes and time, I here summarize the summarized, preliminary work plan to develop a modular, shared, and well documented global, Hager & O'Connell (1981) type, spectral mantle flow tool. Please send me comments on this document. This document specifies the preliminary work plan for the beginning of 2006.

The objective of this project is to provide a commented and tested set of modular source codes to allow the user to compute velocities, stresses, and the geoid for a set of different mechanical boundary conditions and spherical harmonics (SH) based density models. Example applications would be testing other numerical implementations, teaching, estimating the large scale mantle flow field, and coupling with FE codes, for instance.

Besides this standard viscous solution, we will also strive to incorporate visco-elastic relaxation/transient approaches, and later, as a separate package, routines to invert for rigid plate motions. There will be an accompanying package to convert from lon/lat data to SH space, and, within a different effort (basically, an update of Becker & Boschi, 2002, as hosted by the European SPICE network), several global tomographic models will be provided for download.

Immediate goals (January – March 2006)

1. Agree on benchmarks for cross-code comparison and comparison with analytical solutions (see eight suggested initial tests in appendix).
2. Collect existing, working codes (Bernhard, Carolina, and Mark agreed to share their routines). Make those easily compilable, in effect establishing a repository for existing codes. If possible, run benchmarks with existing codes before sharing them with instructions how to do so.
3. Compare Mark's or Carolina's code with rewritten, modular C implementation of Bernhard's code by Thorsten and decide which code to use as a starting point for the community tool.
4. Assemble a working version of the flow code using modular spherical harmonics

implementation and a mix of existing F90 and C code. Strive to have everything in C, for portability, eventually. Desired initial functionality:

- a) Free-slip, no-slip, and plate (SH expansion) surface mechanical boundary conditions.
 - b) Only radial viscosity variations with an arbitrary number of constant viscosity layers.
 - c) Compute incompressible viscous flow solution without phase boundaries and constant gravity throughout mantle robustly up to spherical harmonic degree of 50 or 100.
 - d) Density model can be input as SH parameterization (*e.g.* using Thorsten's, Dahlen & Tromp normalization, format).
 - e) Output of velocities, stresses, and geoid as SH and in lon/lat space.
5. Document and match analytical as well as cross-code benchmark solutions (see appendix). Within a makefile “test” directory, allow the user to compute both analytical benchmark solutions and run Bernhard, Mark's and the new code for comparison with each other. Formulate benchmarks such that CitcomS solutions can be achieved and compared with as well. Benchmarks are eight selected cases from the following ingredients (see appendix):
- a) Use a single analytic, a mid-mantle layer, and two discrete density models, the last three with anomalies up to degrees 2, 20, and 50.
 - b) Use a constant viscosity mantle, and three as well as 13 layers varying viscosity layered models.
 - c) Compute solutions for both no and free slip. Optional: compute solution for prescribed surface velocities to test toroidal flow part.
 - d) Compare velocities and viscous stresses in SH space and on 1 by 1 degree grids at the surface, the CMB, and the mid mantle.
 - e) Compare geoid kernels, and predicted geoid on surface in SH and on 1 by 1 degree grid.

It would be most useful if the codes for the repository could try to incorporate the suggested benchmarks partially, and I will contact those contributing to the benchmarks individually.

6. Provide a complete description/writeup of the theoretical approach within a README type document (many of the published papers contain known typos in the equations). Write results up in a technical paper for publication in G-Cubed.

Planned later extensions:

1. Implement compressibility and phase transitions following Panasyuk *et al.*'s (1996) or Steinberger's approach;

2. Along the line of 1., adjust the depth dependence of g include self-consistent gravitation, or density from PREM.
3. Implement visco-elastic relaxation solution (Love numbers).
4. Provide an external set of routines to solve for rigid plate motions following Ricard & Vigny (1989) following Carolina's or Bernhard's implementation.
5. Implement the iterative solution method for flow in the presence of lateral viscosity contrasts. Implement Steinberger *et al.* (2001) solution for stresses in the lithosphere (as a thin sheet).

Personnell involvement:

Bernhard Steinberger: Contribute flow code with extensions, assist in benchmarking.

Carolina Lithgow-Bertelloni: Possibly contribute flow code, contribute analytical solutions, theory descriptions and benchmarks.

CIG: Possible provide repository and collaboration support.

Clint Conrad: Assist in benchmarking of flow codes and CitcomS comparisons.

Craig O'Neill: Lead programming project, evaluate which starting code to use (Mark's or Thorsten's), implement preliminary community code, run benchmarks, lead in writing paper on project efforts.

Jerry Mitrovica: Share his viscoelastic Love number code.

Mark Richards: Contribute flow code, discuss implementation strategy for visco-elastic relaxation problem with Jerry Mitrovica, assist in benchmarking.

Rick O'Connell: Check/implement a more robust propagator/two point boundary value approach. Assemble benchmark solutions and theory write-ups.

Shijie Zhong: Liaison with CIG for potential repository and programming support.

Thorsten Becker: Assist Craig with coding, write/modify makefiles and portable installation/testing procedures, make sure that library issues are dealt with in a flexible way. Contribute a set of routines reading GMT grd files to convert those into spherical harmonics and back, allowing for plotting of results.

Appendix: Benchmark parameters

In the following, we list three boundary conditions, three viscosity structures, and four density structures, making for 36 potential test cases. We suggest to focus on free slip/no slip, viscosities 3a) and 3b), and density structures 4a), and 4c), making for eight initial test cases.

1. Parameters

In general, use SI units, but cm/yr for velocities, MPa for stresses, and % for density anomalies. For spherical harmonics scalars and poloidal/toroidal fields, use Dahlen & Tromp (1988, B.8) theoretical physics convention or describe how to convert to that format.

<i>Parameter</i>	<i>Value</i>
reference viscosity	10^{21} Pas
radius of Earth	6371 km
gravitational constant	$6.6742 \cdot 10^{-11}$ N m ² /kg ²
gravitational acceleration (constant throughout Earth for simplicity)	10 m/s ²
seconds per year	31556926
average mantle density (constant)	4448.8 kg/m ³
radius of core	3480 km ($r = R(\text{core})/R(\text{earth}) = 0.546225$)
thickness of mantle	2891 km

2. Boundary conditions

CMB is free slip, surface is at least free-slip (a), or no-slip (b). To test toroidal flow solver part, also try to compute velocity and geoid solutions with prescribed surface velocities (c). For plates, use NUVEL-HS2 in NNR reference frame, expanded without tapering up to spherical harmonic degree 127 at

http://geodynamics.usc.edu/~becker/ftp/flow_bench/nnr.nuvel.127.pt.ab.gz

in units of cm/yr and Dahlen & Tromp (1998) format, listed in

A_{lm} B_{lm} C_{lm} D_{lm}

where A, B are poloidal, and C, D toroidal flow components (see, *e.g.*,

<http://geodynamics.usc.edu/~becker/tomography/node12.html>)

3. Viscosity structures

- a) constant throughout mantle at reference value 1 (times reference viscosity)
- b) Upper 100 km ($\leq 100\text{km}$) and lower mantle ($> 660\text{km}$) 100, upper mantle 1 times reference viscosity.
- c) 13 layers as specified in the file at

http://geodynamics.usc.edu/~becker/ftp/flow_bench/visco.b3

Format: first line: nd #

of data entries.

Following nd lines:

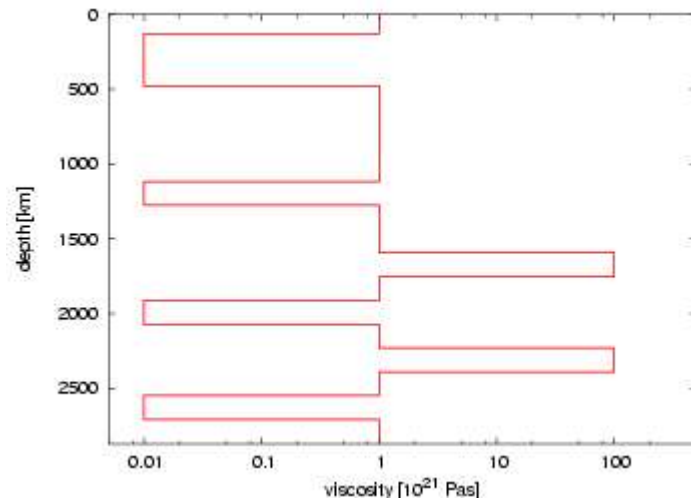
R/R_{earth} η/η_{ref}

such that the 13

viscosity layers work

out to look like figures

on right.



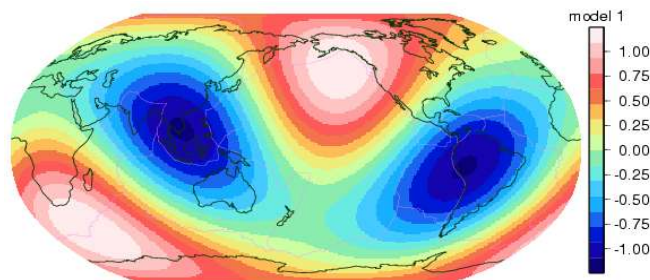
4. Density structures

All discrete models are given as spherical harmonics expansions in Dahlen & Tromp (1998) convention on 58 evenly, 50 km spaced layers from 2870 km to 20 km depth throughout the mantle using the file format convention as described on

<http://geodynamics.usc.edu/~becker/tomography/node12.html>

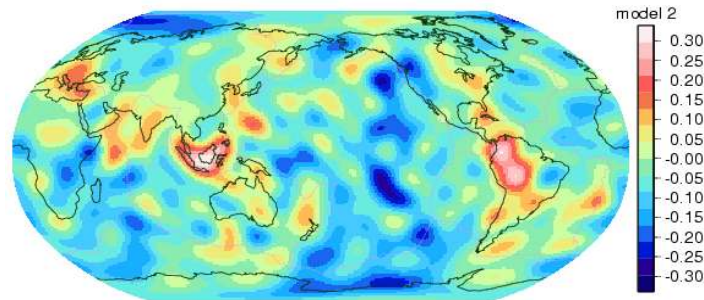
The test models are available for download and are given by:

- a) Zhong *et al.*'s (2000) first appendix C1 benchmark with a single layer at mid mantle depth. Only for this test, use Shijie's viscosity structures and inner core radius of $R(\text{core})/R(\text{earth}) = 0.55$ instead of 0.546225. More specifically



(paraphrased from Shijie's comments, equation numbers refer to Zhong *et al.*, 2000): The equations that were solved were the non-dimensional conservation equations (*i.e.* eqs. 6 & 7) where the driving force is represented by $X Ra T$, where X and Ra are defined in eq. 9. For this benchmark, T is given in equation C1 (*i.e.*, a delta function in radial position at the mid-mantle depth multiplied with a spherical harmonic function of some l and m), and $X Ra = 1$. This use of a delta function here is similar to those in kernel calculations by Richards and Hager (1984) and Hager and Richards (1989). If you use density in eq. 2, then $X Ra T$ is $d\rho/g$. Notes: (I) On the implementation of the delta function in the analytic solution and in the finite element method: For the analytics, the delta function in radial direction really simplifies the solutions (Hager and O'Connell, 1981, talked about this). For finite elements, we let T be zero everywhere except on one layer of nodes (*i.e.*, at $r=(r_o+r_i)/2$ or mid-mantle) where T is equal to $(1/dr)*Y(l,m)$, where dr is the radial grid spacing or $(r_o-r_i)/nel_r$ with nel_r as the number of element in radial direction. (II) There might be a minor error in the description of this benchmark in appendix C of Zhong *et al.* (2000), where it is stated that $Ra=1$ not $X Ra=1$ as stated above because of technical changes in the revisions.

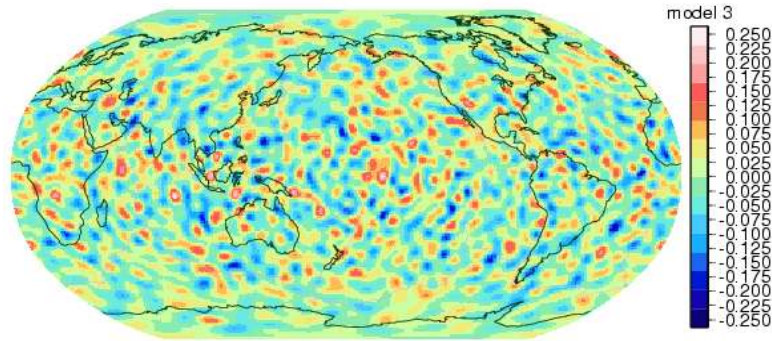
- b) A degree 2 pattern whose amplitude oscillates with two sine functions throughout the mantle. The density anomaly is shown below at 1020 km depth, units are %, and the coefficients are available at http://geodynamics.usc.edu/~becker/ftp/flow_bench/d1.m.ab.gz
- c) The 3rd model is Ritsema and van Heijst (2000) S20RTS seismic tomography model scaled by 0.2, re-parameterized and interpolated to the same depth levels as model 1. At 1020 km, this model should look like so:



and the coefficients are at

http://geodynamics.usc.edu/~becker/ftp/flow_bench/d2.m.ab.gz

- d) The 4th model uses a random, white noise spectrum, looks like plotted below at 1020 km, and can be downloaded at http://geodynamics.usc.edu/~becker/ftp/flow_bench/d3.m.ab.gz



4. Model evaluation

- a) Evaluate velocities at surface, CMB, and 1020 km depth in spherical harmonics for the three density cases, all formally up to degree and order 50, Dahlen & Tromp (1998) poloidal/toroidal format. Evaluate velocities at same depth on 1 by 1 degree grid going from 0 to 359 lon, and -89.5 to 89.5 latitude.
- b) Evaluate geoid kernels, and surface geoid in meters, spherical harmonics. For the geoid, assume an incompressible core and mantle and use a constant gravity of 10 m/s² for simplicity.
- c) If code allows for it, evaluate stresses τ_{rr} , $\tau_{r\theta}$, $\tau_{r\phi}$, $\tau_{\theta\theta}$, $\tau_{\theta\phi}$, $\tau_{\phi\phi}$ in MPa at same layers, where r , θ , ϕ are the regular spherical coordinates radius (unit vector up), co-latitude (unit vector South), and longitude (unit vector East).

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