

Reservoir Simulation Magma Dynamics

Marc Hesse



[Outline

1. **Introduction**
 1. Reservoir Simulation (RS)
 2. RS vs. Magma Dynamics (MD)
2. **Numerical Issues**
 1. Permeability Heterogeneity
 2. Permeability Anisotropy
 3. Grid Adaptivity
 4. Multiscale Methods (MSM)
3. **Summary**
(High-Resolution/Order Schemes)

Idealized RS Equations I

matrix is rigid $\Rightarrow \phi = \phi(\bar{x})$, wetting & non-wetting phase

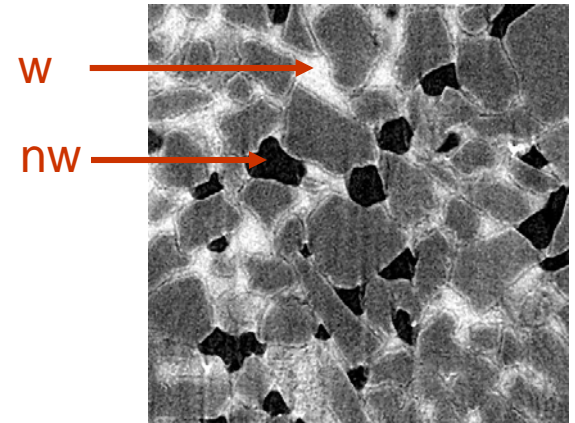
$$\text{ saturations: } S_p = \varphi_p / \phi \quad S_w + S_{nw} = 1$$

$$\text{ saturation eqn.: } \phi \frac{\partial S_w}{\partial t} + \nabla \cdot \bar{u}_w = \Gamma_w$$

$$\text{ Darcy's law: } \bar{u}_p = -\lambda_p (\nabla p_p + \rho_p \bar{g})$$

$$\text{ mobility \& rel. perm.: } \lambda_p = \frac{K(\bar{x})}{\mu_p} k_{r,p}(S_w) \quad k_{r,p} = S_p^2$$

$$\text{ capillary pressure: } P_c(S_w) = p_{nw} - p_w$$



Benson et al. 06

Idealized RS Equations II

assuming $P_c = 0$ and $\nabla \cdot (\bar{\mathbf{u}}_w + \bar{\mathbf{u}}_{nw}) = 0$ introducing

the fractional flow $f_w(S_w) = \frac{\bar{\mathbf{u}}_w}{\bar{\mathbf{u}}_T} = \frac{\lambda_w}{\lambda_T} \left[1 - \frac{k_{r,nw}}{\bar{\mathbf{u}}_T} N_g \bar{\mathbf{n}}_z \right]$

where $\lambda_T = \lambda_w + \lambda_n$ and $\bar{\mathbf{u}}_T = \bar{\mathbf{u}}_w + \bar{\mathbf{u}}_{nw} = -\lambda_T \nabla p$

saturation equation: $\phi \frac{\partial S_w}{\partial t} + \bar{\mathbf{u}}_T \cdot \nabla f_w = \Gamma_w$

pressure equation: $\nabla \cdot [\lambda_T(S) \nabla p] = \sum_p (\Gamma_p - N_g \bar{\mathbf{g}} \cdot \nabla \lambda_p)$

[Comparison RS vs. MD]

RS

MD

hyperbolic: $\phi \frac{\partial S_w}{\partial t} + \vec{u}_T \cdot \nabla f_w(S_w) = \Gamma_w$

$$\frac{\partial \phi}{\partial t} + \vec{v}_s \cdot \nabla \phi = \frac{P}{\xi} + \Gamma_w$$

elliptic: $\nabla \cdot [\lambda_T \nabla p] = R$

$$\nabla \cdot (k_\phi \nabla P) - \frac{P}{\xi} = -\nabla k_\phi \cdot \vec{g}$$

open problem: **Dynamics of 2 fluids in compacting matrix**

- applicable to:
- 1) flux melting in subduction zones
 - 2) hydrates in soft sediments
 - 3) brine – hydrocarbon – salt dynamics

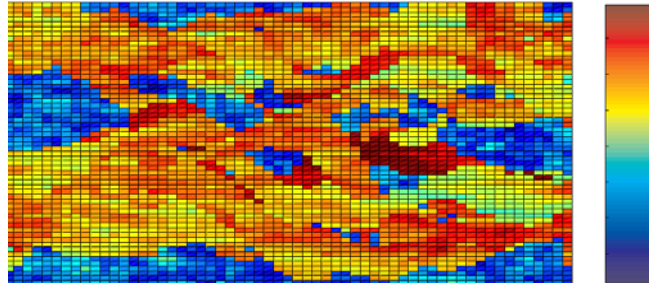
interesting maths problem: $\lim_{\xi \rightarrow 0} \Rightarrow$ **quasi-linear hyperbolic system**

[Heterogeneity in RS]

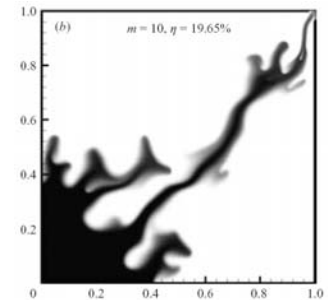
pressure equation $\nabla \cdot (\lambda_T(x) \nabla p) = R$



Jasper National Park



bottom layer SPE 10 (Aarnes et al. 05)



Riaz & Meiburg 04

lithological heterogeneity: preexisting

$$\text{Var}(\lambda_T) = 10^8$$

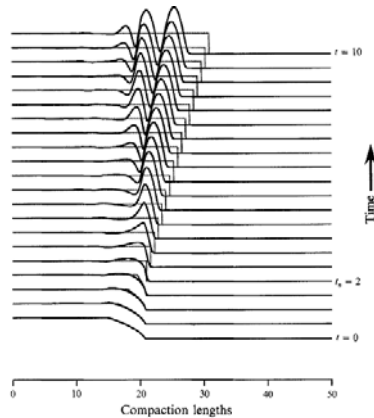
dynamic heterogeneity: mobility (viscosity)

$$\text{Var}(\lambda_T) = 10^2$$

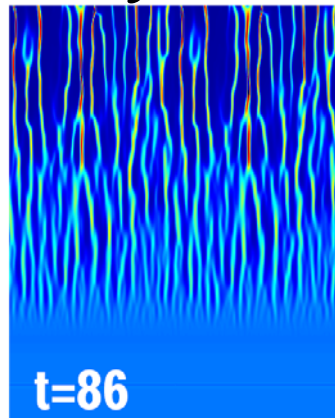
viscous fingering

[Heterogeneity in MD]

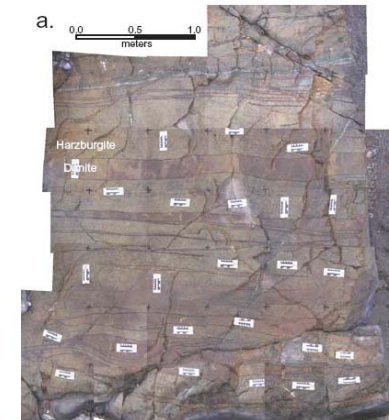
Helmholtz eqn:
$$\nabla \cdot (k_\phi \nabla P) - \frac{P}{\xi} = -\nabla k_\phi \cdot \bar{g}$$



Spiegelman 93



Spiegelman et al. 01



Braun & Kelemen 02

lithological heterogeneity: preexisting ?, generated during melt extraction
 dynamic heterogeneity: (de)compaction \Rightarrow solitary waves
 reactive infiltration instability \Rightarrow channels

[Heterogeneity in RS & MD]

Reservoir Simulation

preexisting het. dominant:

- ⇒ pervasive
- ⇒ all length scales
- ⇒ random component
- ⇒ long correlation length

essentially stationary

discontinuous

$$\text{Var}(\lambda_T) = 10^8 (10^{12})$$

Magma Dynamics

focus on dynamic het.:

- ⇒ localized to solution
(solitons & channels)
- ⇒ dynamic length scales

transient

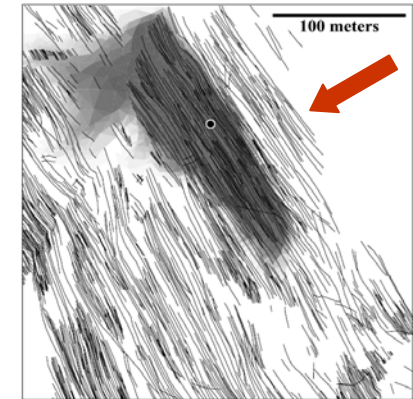
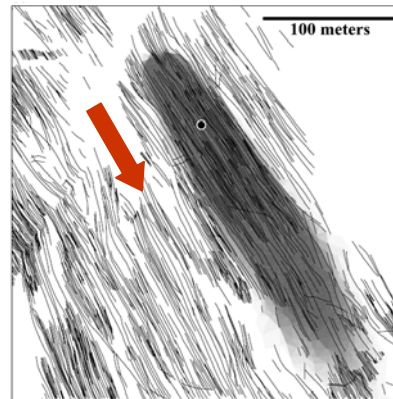
continuous & discontin.

$$\text{Var}(k_\phi) = 10^4$$

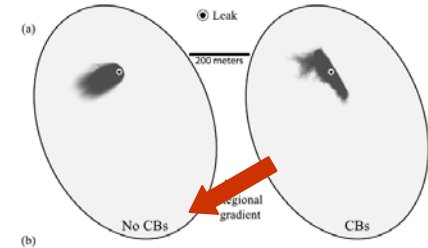
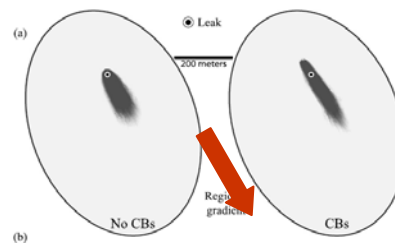
Is preexisting lithological heterogeneity important in MD?

[Permeability Anisotropy in RS]

direct simulation:
Sternloff et al. (2006)



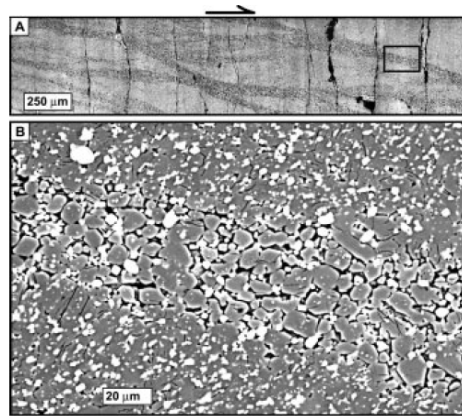
reservoir scale:



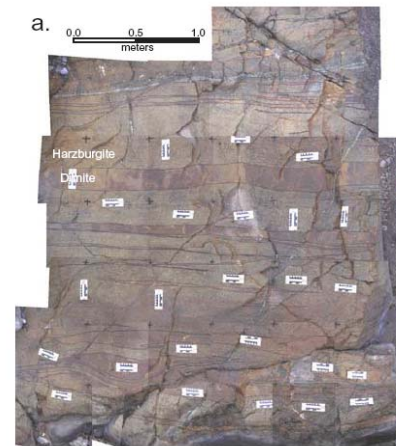
⇒ large natural anisotropies (1:1000)

$$K = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}$$

Origins of Anisotropy in MD



Holtzman et al. 03



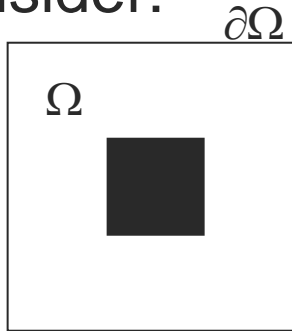
Braun & Kelemen 02

What is the magnitude of this anisotropy?

Anisotropy dynamically related to deformation of matrix!

[Monotonicity & Anisotropy]

consider:

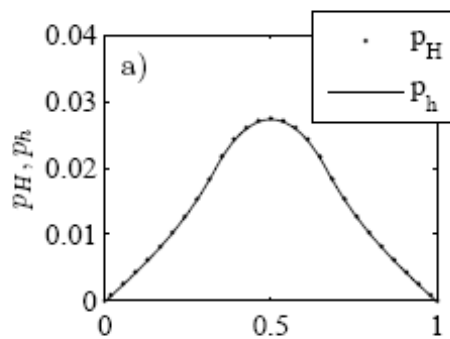


$$-\nabla \cdot (\lambda_T \nabla p) = R(x) \quad x \in \Omega \quad \text{Hesse et al. 08}$$

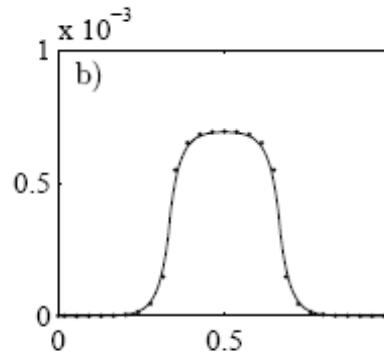
$$p(x) = 0 \quad x \in \partial\Omega$$

$$\lambda_T = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \quad R(\bar{x}) = \begin{cases} 1 & x \in \blacksquare \\ 0 & x \in \square \end{cases}$$

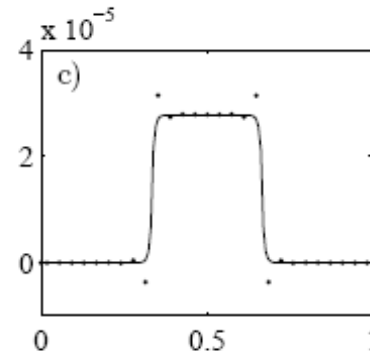
Hopf's lemma: **If $R(x) \geq 0$ in Ω , then p has no minimum in Ω .**



$b/a = 1$



$b/a = 100$



$b/a = 2500$

Discretization for Anisotropy

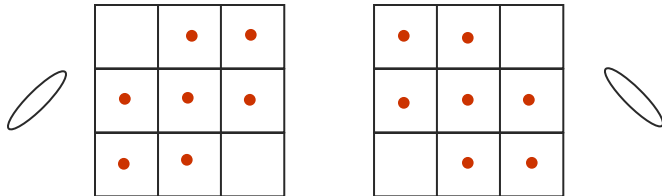
Nordbotten et al. 07

Discrete analog of Hopf's lemma conditions monotonicity of stencil

- quadrilateral grid
- conservative
- 9-point stencils
- 1st order or higher

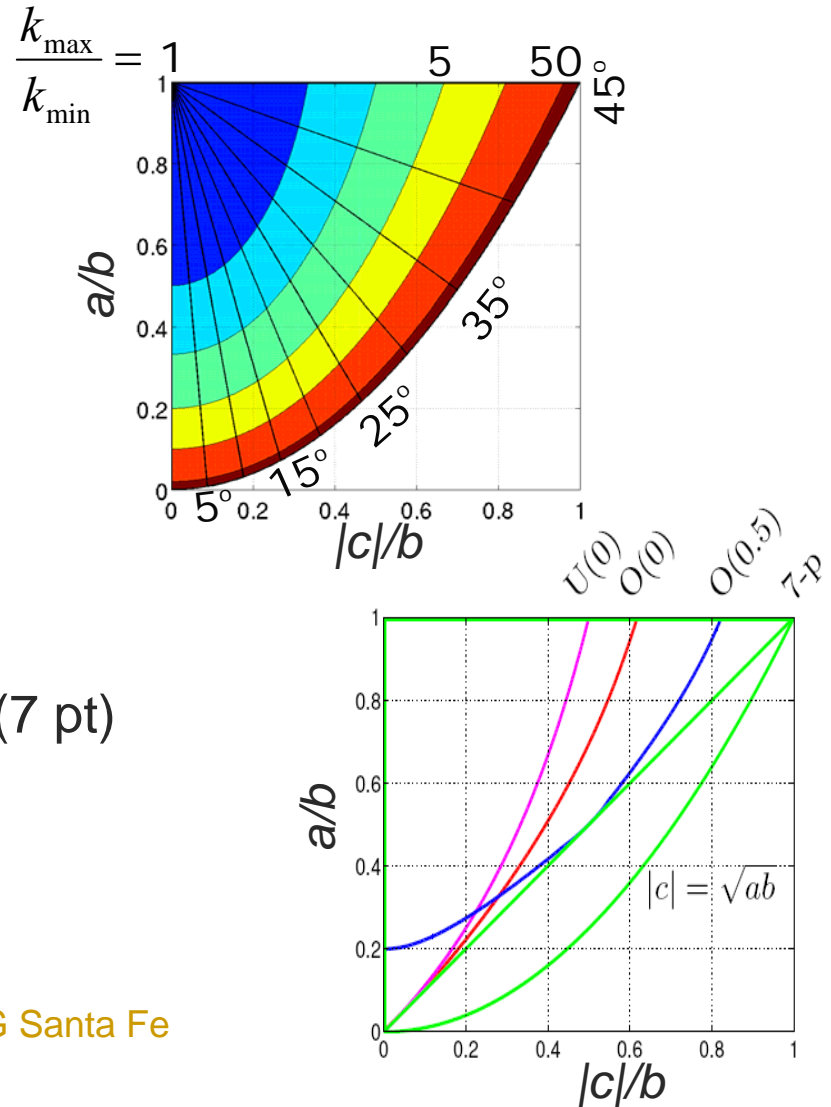
⇒ monotonicity regions

stencil with optimal monotonicity (7 pt)



09/15/2008

CIG Santa Fe



What does this mean for MD?

Hopf's lemma \Rightarrow identify numerical artifacts in Poisson eqn
Similar properties for Helmholtz eqn? \Rightarrow more difficult

Sign change ($\phi < 0$) due to numerical artifacts \Rightarrow blow up
Helmholtz eqn. more sensitive than Poisson eqn.?

\Rightarrow Caution is necessary with anisotropy!

Use a stencil that is monotone for Poisson, hope for best!

Open research problem for a numerical analyst!

Adaptivity in Heterogenous Media

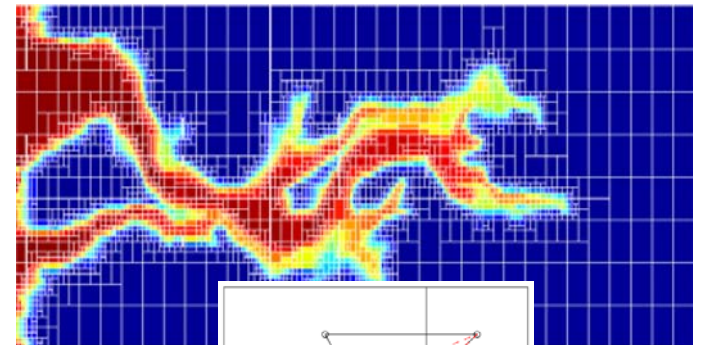
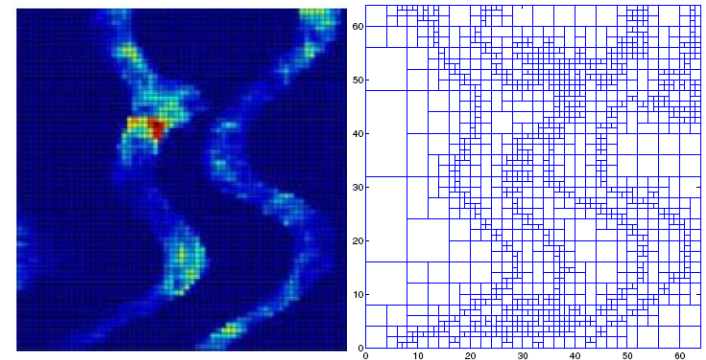
Gerritsen & Lambers 08

Cartesian Adaptive Refinement:

- data structures simple & efficient
 - anisotropic refinement
- 1) static refinement (difficult)
channels, wells, faults
 - 2) dynamic refinement
saturation fronts

Problems:

hanging nodes \Rightarrow unstructured method DG
effective parameters \Rightarrow multiscale methods



[Adaptivity RS vs. MD]

Reservoir Simulation

heterogeneity:

at all length scales

strong random component

coarse operator:

complicated \Rightarrow multiscale

adaptivity criteria:

complicated (edge detection)

discretization:

FV \Rightarrow hanging nodes

Magma Dynamics

localized to solution
solitons & channels

simpler

simpler, based on solution

unstructured DG, Spectr. Elem,

Adaptive gridding simpler and more promising for MD than RS.

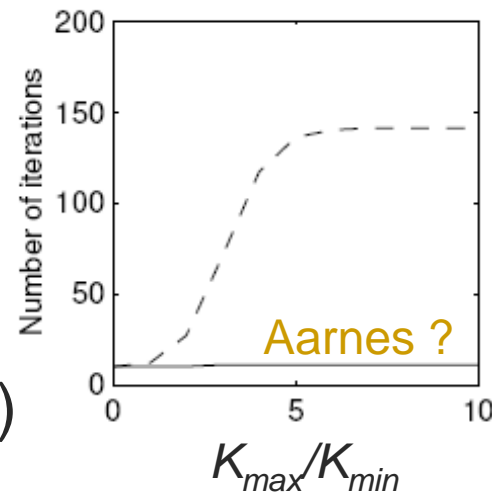
Multiscale Methods (MsM)

Poisson equation: $\nabla \cdot [\lambda_T(x) \nabla p] = R$

Solve eqn on coarse grid, and reconstruct fine solution.

Problems:

- 1) no scale separation
⇒ sample entire field, $O(N)$
- 2) field is transient
⇒ iterative solvers (MG, NODD)



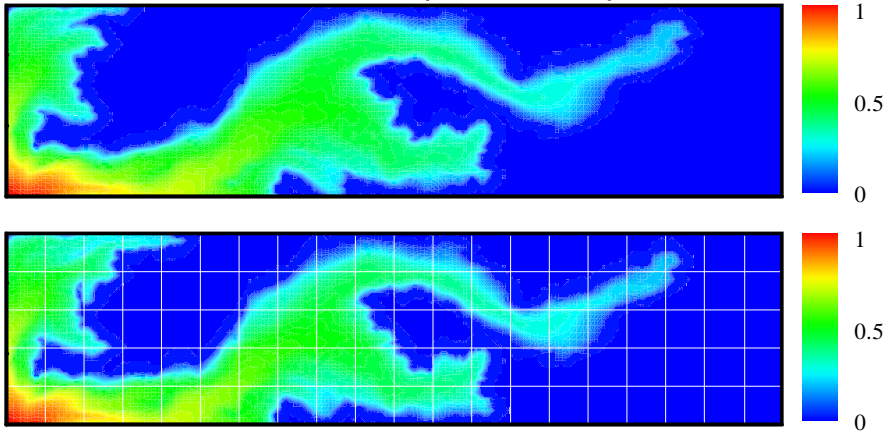
MG and DD: simple coarse operators and iterate

MS: complex coarse operator and don't iterate

Multiscale Methods (MsM)

Is error is acceptable?

reference (220x55)

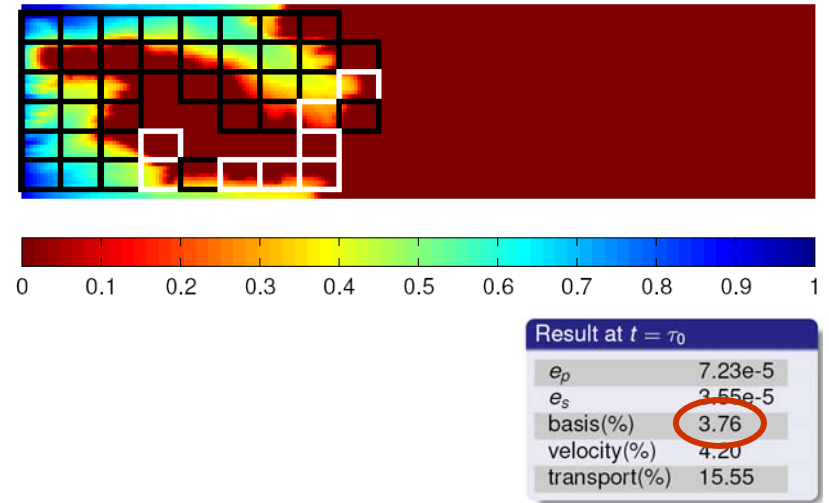


MSFV (20x5)

Hadjibeygi et al. 08

Yes, most of the time.

Is the cost competitive?

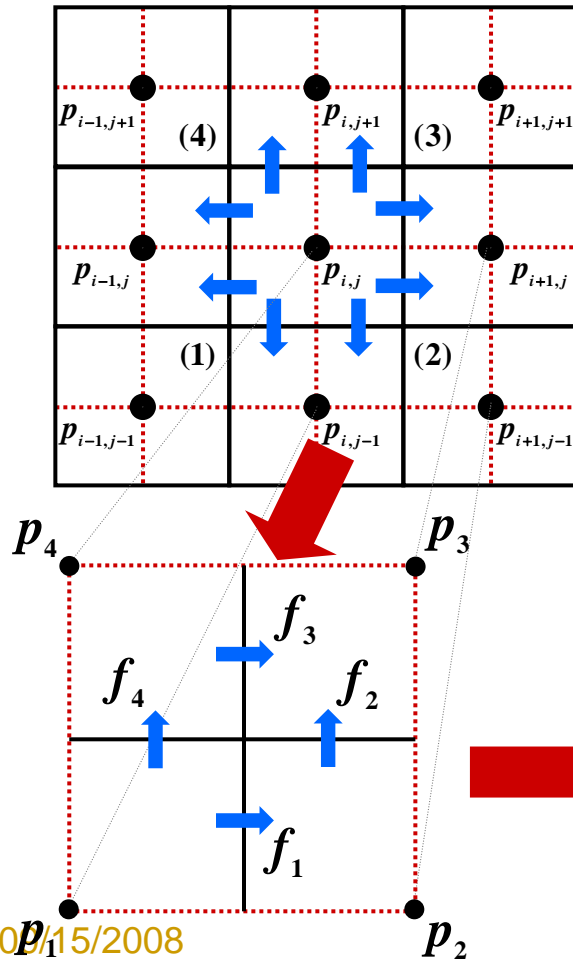


Zhou et al. (in prep.)

“Yes”, through adaptivity.

Finite Volume Formulation

Jenny et al. 03, Ginting 04, Hesse et al. 08



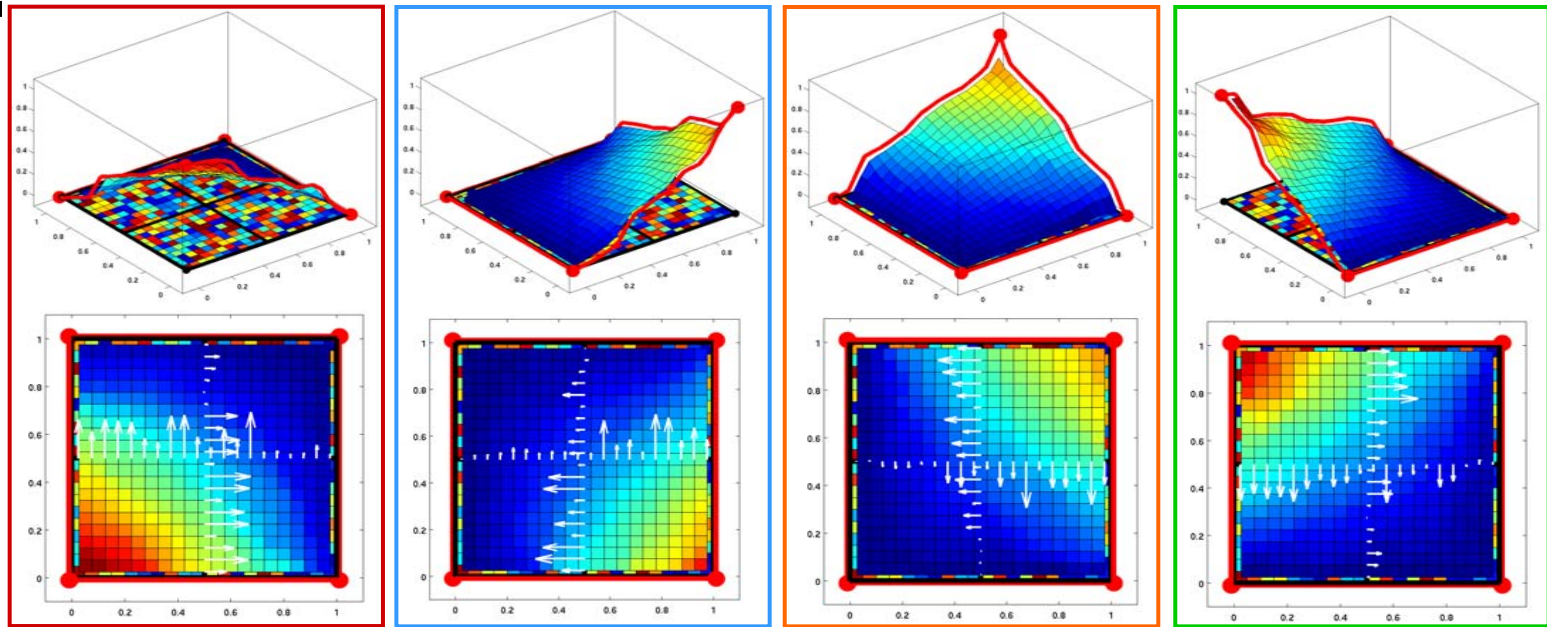
□ control volume
 □ interaction region

$$L_h p_h = \begin{bmatrix} m_{-1,1} & m_{0,1} & m_{1,1} \\ m_{-1,0} & m_{0,0} & m_{1,0} \\ m_{-1,-1} & m_{0,-1} & m_{1,-1} \end{bmatrix}_h p_h$$

$$m_{1,0} = -t_{1,2}^{(3)} - t_{4,2}^{(3)} + t_{4,3}^{(2)} - t_{3,3}^{(2)}$$

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

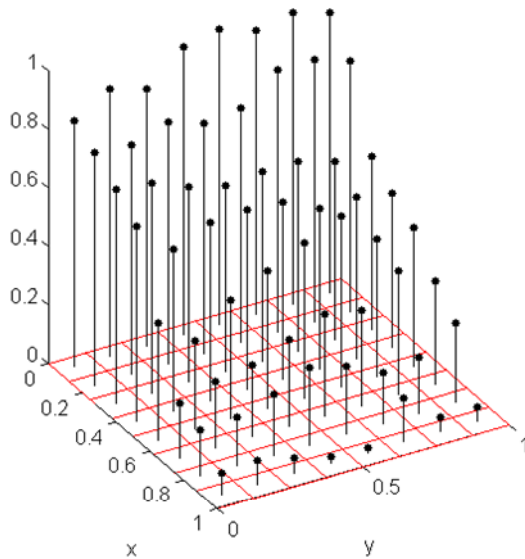
Multiscale Methods RS



$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

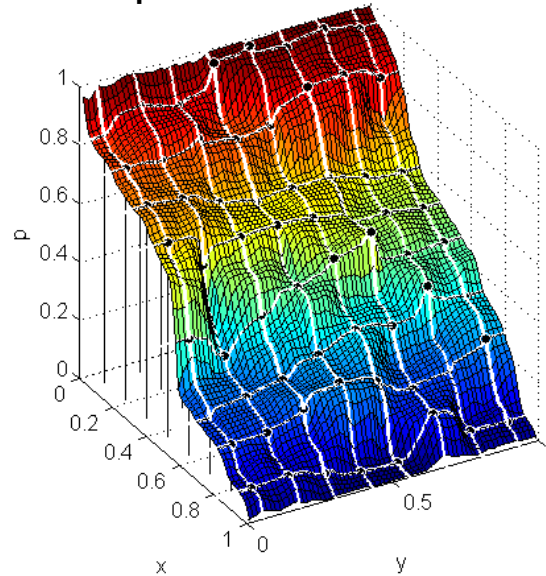
3 Pressure Fields

Coarse Pressure:



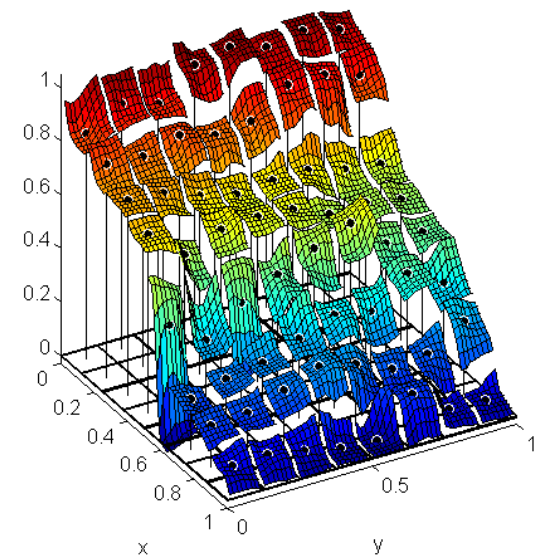
monotonicity not
guaranteed

Interpolated Pressure:



fine p continuous
fine flux discontinuous

Conservative Pressure:



fine p discontinuous
fine flux continuous

continuous flux is essential \Rightarrow accept discontinuous pressure

[Iterative MsM]

Need error control for MS methods!

global information improve bc's basis functions

ALG (Chen 03) ALG-MsM (Durlafsky et al. 07)

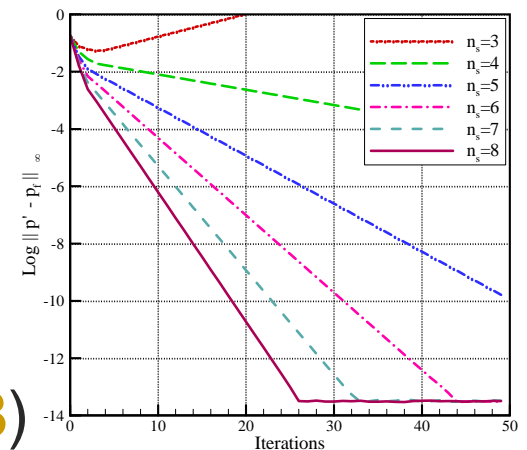
⇒ iterations, but not a solver

MsM extended to full solver:

Iterative MsM (Hadjibeygi et al. 08)

Solvers with Ms ideas:

NODD-MsM (Nordbotten & Bjorstad 08)



[MS for MD]

$$\nabla \cdot (k_\phi \nabla P) - \frac{P}{\xi} = -\nabla k_\phi \cdot \vec{g}$$

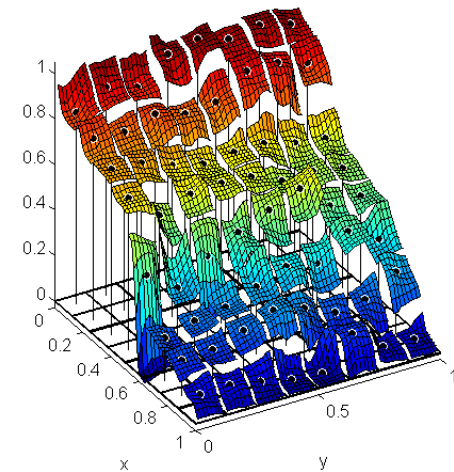
continuous flux field ∇P

discontinuous pressure P

⇒ instability

large perturbation of coefficients by dynamics

⇒ need to update often, expensive



Multiscale simpler and more promising for RS than MD.

[Summary]

1. Governing equations
 - ⇒ Poisson vs. Helmholtz
 - ⇒ Combine RS and MD to look at 3-phase flow
2. Nature of heterogeneity is fundamentally different
 - ⇒ adaptive gridding in MD
 - ⇒ multiscale methods in RS
 - ⇒ higher order methods more impact in MD
3. Anisotropy MD can benefit from work done in RS
 - ⇒ dynamic anisotropy in MD challenging & novel
 - ⇒ discretizations for anisotropic Helmholtz eqns

References 1

- Benson, Tomutsa, Silin, and Kneafsey (2006) Core Scale and Pore Scale Studies of Carbon Dioxide Migration in Saline Formations, *GHGT-8*, Trondheim, Norway, June 2006
- Aarnes, Kippe, and Lie (2006) Mixed multiscale finite elements and streamline methods for reservoir simulation of large geomodels, *Water Resour. Res.*, **28**(3), 257-271
- Riaz and Meiburg (2004) Vorticity interaction mechanisms in variable-viscosity heterogeneous miscible displacements with and without density contrast, *J. Fluid Mech.*, **517**, 1–25
- Braun and Kelemen (2002) Dunite distribution in the Oman Ophiolite: Implications for melt flux through porous dunite conduits, *Geochem. Geophys. Geosyst.* , **3**(11), 8603
- Spiegelman, Kelemen and Aharonov (2001) Causes and consequences of flow organization during melt transport: The reaction infiltration instability in compactible media, *J. Geophys. Res.*, **106**, 2061-2077
- Spiegelman (1993) Flow in deformable porous media. Part 2 Numerical analysis - the relationship between shock waves and solitary waves, *J. Fluid Mech.*, **247**, 3-63
- Sternlof, Karimi-Fard, Pollard, and Durlofsky (2006) Flow and transport effects of compaction bands in sandstone at scales relevant to aquifer and reservoir management, *Water Resour. Res.*, **42**, W07425
- Holtzman, Kohlstedt, Zimmerman, Heidelbach, Hiraga, and Hustoft (2003) Melt Segregation and Strain Partitioning: Implications for Seismic Anisotropy and Mantle Flow, *Science*, **301**, 1227-1230

References 2

- Hesse, Mallison, and Tchelepi (2008) Compact multiscale finite volume method for heterogeneous anisotropic elliptic equations, *Multiscale Model. Simul.*, **7**(2),934-962.
- Nordbotten, Aavatsmark, and Eigestad (2007) Monotonicity of control volume methods, *Numer. Math.*, **106**, 255-288
- Gerritsen and Lambers (2008) Integration of local-global upscaling and grid adaptivity for simulation of subsurface flow in heterogeneous formations, *Comput Geosci*, **12**(2),193-208
- Hajibeygi, Bonfigli, Hesse, and Jenny (2008) Iterative multiscale finite-volume method, *J. Comput Phys*, **227**, 8604-8621
- Jenny, Lee, and Tchelepi (2003) Multi-scale finite-volume method for elliptic problems in subsurface flow simulation, *J. Comput Phys*, **187**, 47-67
- Ginting (2004) Analysis of two-scale finite volume element method for elliptic problem, *J. Numer. Math.*, **12**, 119-141.
- Chen, Durlofsky, Gerritsen, and Wen (2003) A coupled local-global upscaling approach for simulating flow in highly heterogeneous formations, *Adv. Water Resour.*, **26**, 1041-1060
- Durlofsky, Efendiev, Ginting (2007) An adaptive local-global multiscale finite volume element method for two-phase flow, *Adv. Water Resour.*, **30**, 576–588
- J. M. Nordbotten · P. E. Bjørstad (2007) On the relationship between the multiscale finite-volume method and domain decomposition preconditioners, *Comput Geosci*, **12**, 367–376